Nonlinear AHE

Introduction of Nonlinear AHE

The anomalous Hall conductivity of an electron system whose Hamiltonian is invariant under time-reversal symmetry is forced to vanish. Crystals with sufficiently low symmetry can have resistivity tensors which are anisotropic, but Onsager's reciprocity relations force the conductivity to be a symmetric tensor in the presence of time-reversal symmetry. Hence, when the electric field is along its principal axes, the current and the electric field are collinear, at least to the first order in electric fields. However, this constraint is only for the linear response and fails at the higher order of the electric field.

Requirement

In your presentation, you are asked to derive the response of a constant electronic field to seconder order and calculate the anomalous Hall current. Also, you should evaluate it in a model system with appropriate symmetries.

Derivation of Response Current

We consider the Bloch electron dynamics in a single-band system for simplicity but allow it to be two or three dimensions. The equations of motion are

$$v_a = \partial_a \epsilon(k) + \varepsilon_{\mathrm{abc}} \Omega_b \dot{k}_c$$

$$\dot{k}_c = -eE_c$$

When applying a constant electric field E_c, you should consider the Boltzmann equation under relaxation time approximation and derive the distribution function $f(k)$ up to the second order of the electric field.

The response current is given by $j_a = -e \int_k f(k) v_a$, evaluate it up to the second order of electric field.

Symmetrical analysis

Write the current in the form $j_a = \chi_{\mathrm{abc}} \mathcal{E}_b \mathcal{E}_c$, and obtain the following form of the response coefficient at the first order of the relaxation time

$$\chi_{\mathrm{abc}} = \varepsilon_{\mathrm{adc}} \frac{e^3 \tau}{2} \int_k (\partial_b f_0) \Omega_d$$

Under a point-group symmetry operation, find the transformation law of the response coefficient. If the crystal has C_6 in 2-D hexagon crystal and D_{4h} groups in 3-D fcc crystal, find the form of the response coefficient (which elements of the response coefficients can be nonzero).

Evaluation in a model Hamiltonian

Consider the following model Hamiltonian,

$$H_{s\Lambda} = v_x k_x \sigma_y - s v_y k_y \sigma_x + s \alpha k_y + \beta \sigma_z$$

where $s = \pm 1$. β is the size of the gap opened by the ferroelectric distortion.

Evaluate the response coefficient in this system and plot how it depends on the parameter in the effective Hamiltonian.

Reference

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[3] https://press.princeton.edu/books/hardcover/9780691162690/group-theory-in-a-nutshell-for-physicists
[4] https://www.nature.com/articles/s41563-019-0294-7